Introduction to Self-stabilization

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Outline

Example

- $U_0 = a$
- $U_{n+1} = \frac{U_n}{2}$ if U_n is even
- $U_{n+1} = \frac{3U_n+1}{2}$ if U_n is odd

Example

- $U_0 = a$
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n	0	1	2	3	4	5	6	7	8	9	10	11
U_n	7	11	17	26	13	20	10	5	8	4	2	1

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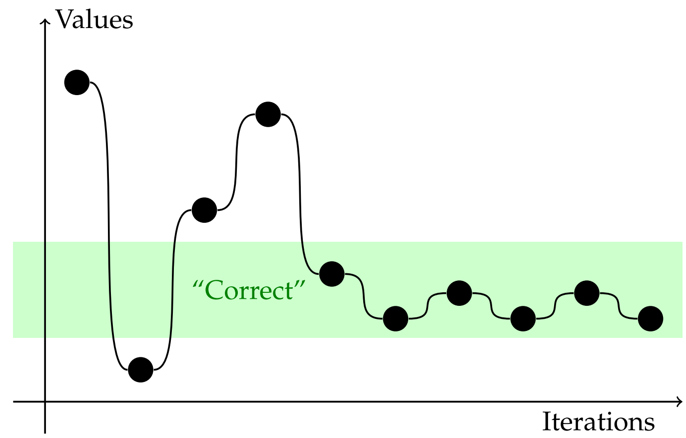
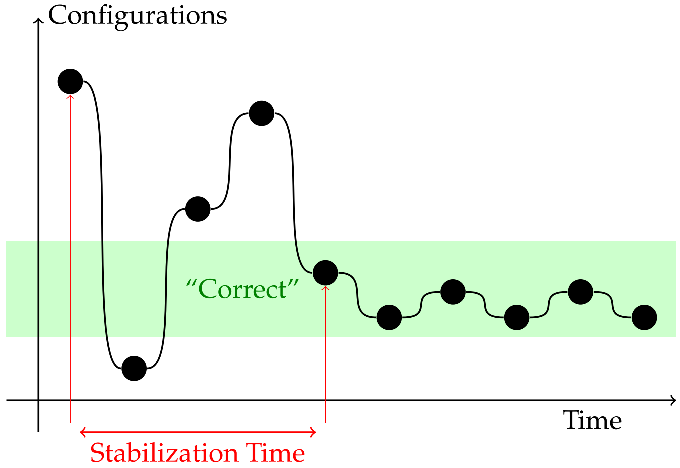
n	0	1	2	3	4	5	6	7	8	9	10	11
U_n	7	11	17	26	13	20	10	5	8	4	2	1

- Converges towards a "correct" behavior
  - 1212121212121212121212121212...
  - Independent from the initial value

Example



Self-stabilization

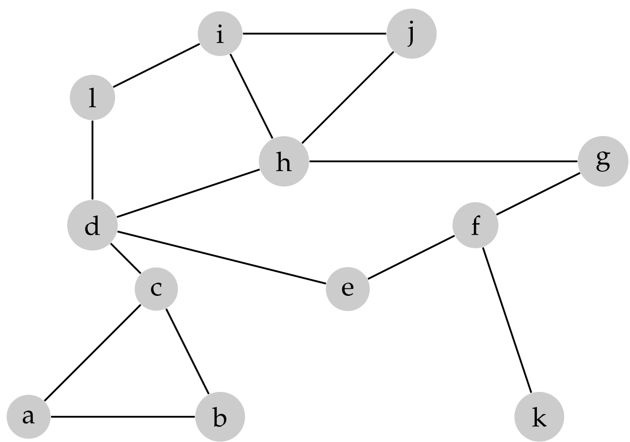


Memory Corruption

- Example of a sequential program:

```
int x = 0;
...
if( x == 0 ) {
  // code assuming x equals 0
}
else {
  // code assuming x does not equal 0
}
```


Distributed Systems



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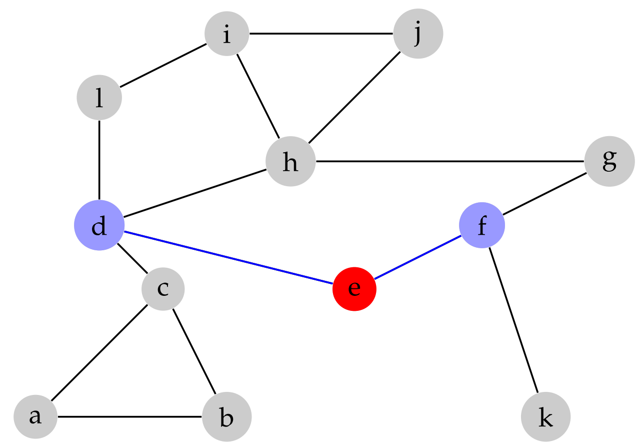

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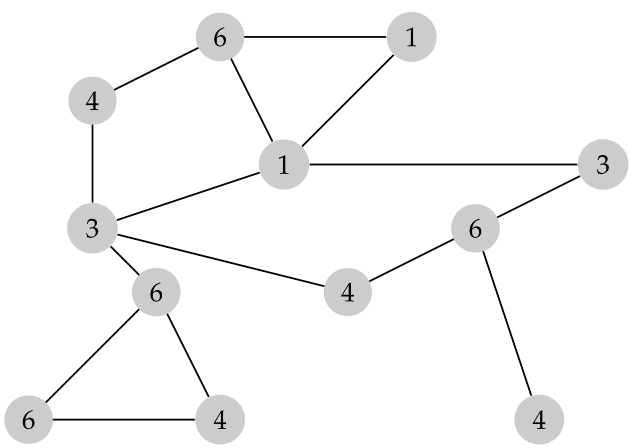

Distributed Systems

- Locality of information

Distributed Systems

- Locality of information
- Locality of time

Distributed Systems

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- ⇒ non-determinism

Distributed Systems

- Locality of information
- Locality of time
- ⇒ non-determinism

Definition (Configuration)

Product of the local states of the system components.

Definition (Execution)

Interleaving of the local executions of the system components.

Distributed Systems

Definition (Classical System, *a.k.a.* Non stabilizing)

Starting from a particular initial configuration, the system immediately exhibits correct behavior.

Definition (Self-stabilizing System)

Starting from any initial configuration, the system eventually reaches a configuration from with its behavior is correct.

Self-stabilization

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- defined by Dijkstra in 1974

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Starting from any initial configuration, the system eventually reaches a configuration from with its behavior is correct.

- defined by Dijkstra in 1974
- advocated by Lamport in 1984 to addesss fault-tolerant issues

Atomicity

- Example of “stabilizing” sequential program

```
int x = 0;
...
while( x == x ) {
  x = 0;
  // code assuming x equals 0
}
```


Atomicity

- Example of “stabilizing” sequential program

```
0 iconst_0
1 istore_1
2 goto 7
5 iconst_0
6 istore_1
7 iload_1
8 iload_1
9 if_icmpeq 5
```


Atomicity

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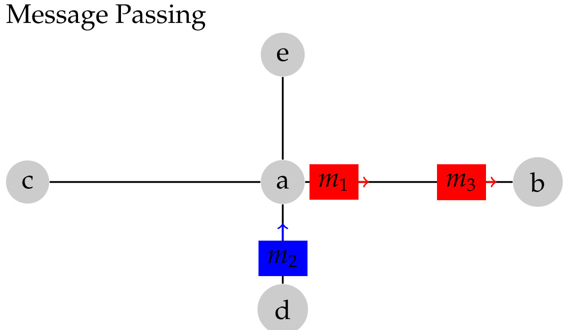
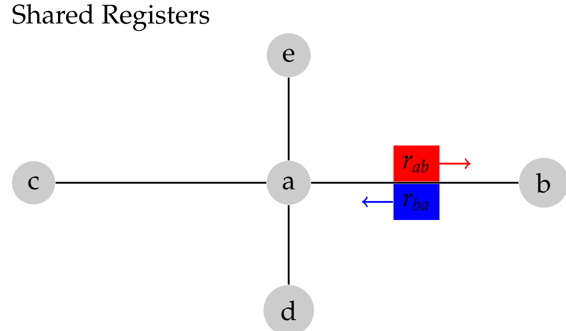
Communications

- Message Passing



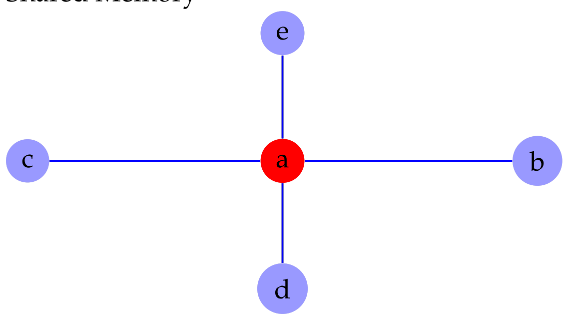
Communications

- Shared Registers



Communications

- Shared Memory



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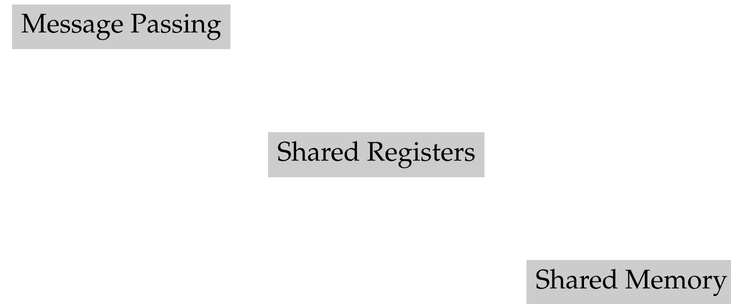

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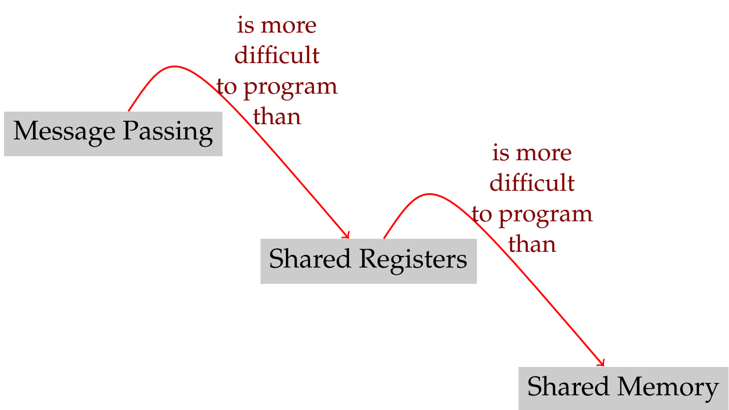

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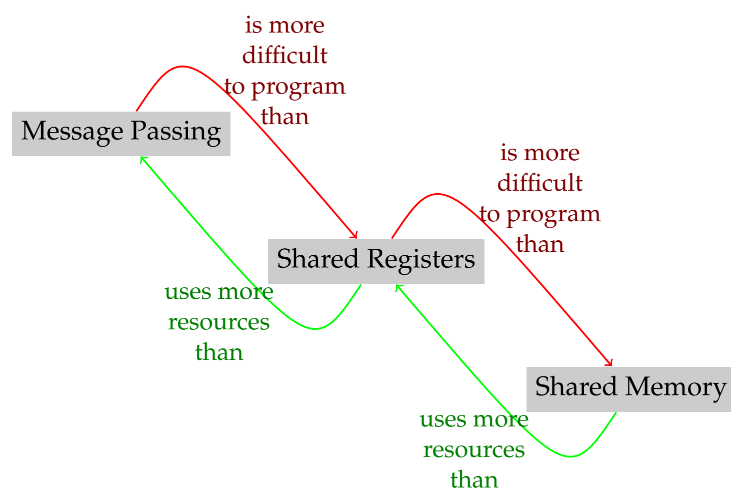

Example

Definition (Shared Memory)

In one atomic step, read the states of all neighbors and write own state

Definition (Guarded command)

- Guard → Action

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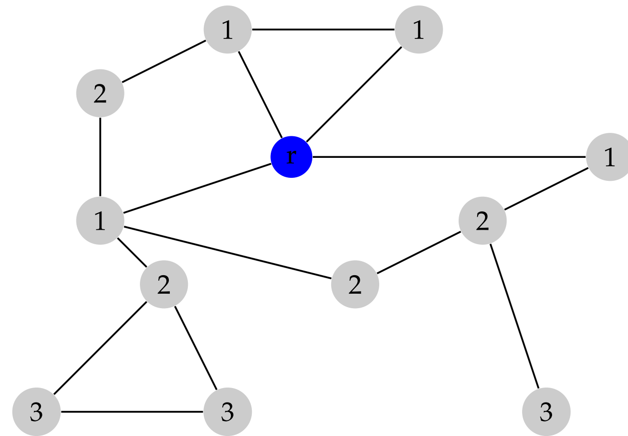
Definition (Shared Memory)

In one atomic step, read the states of all neighbors and write own state

Definition (Guarded command)

- Guard → Action
- Guard: predicate on the states of the neighborhood
- Action: executed if *Guard* evaluates to true

Example



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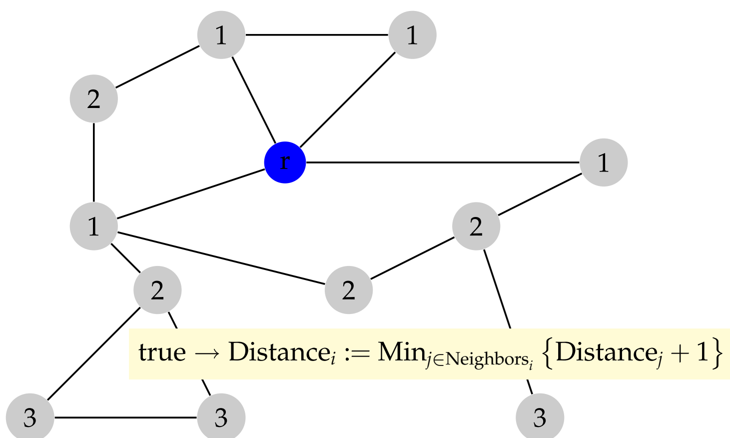

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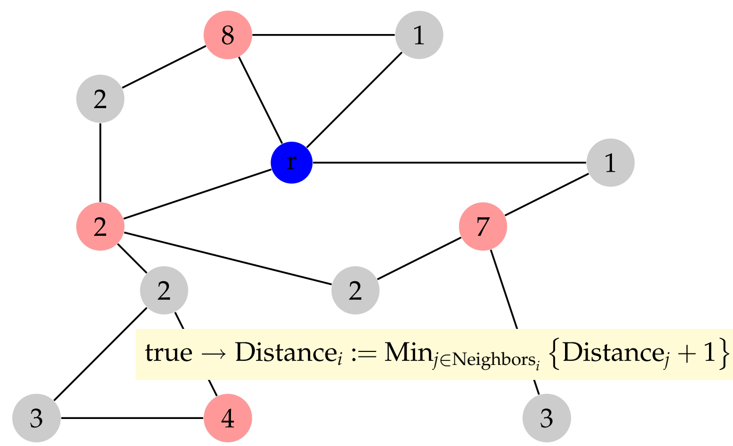

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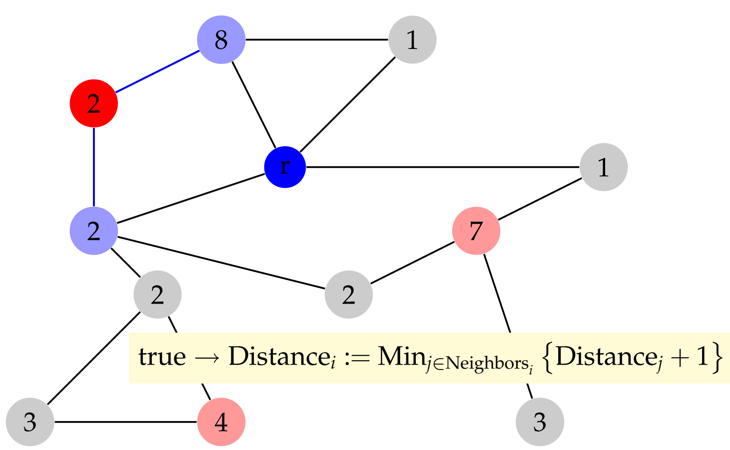

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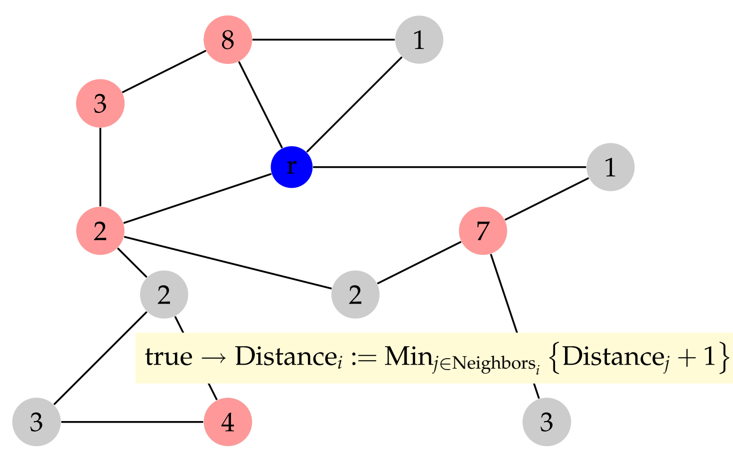

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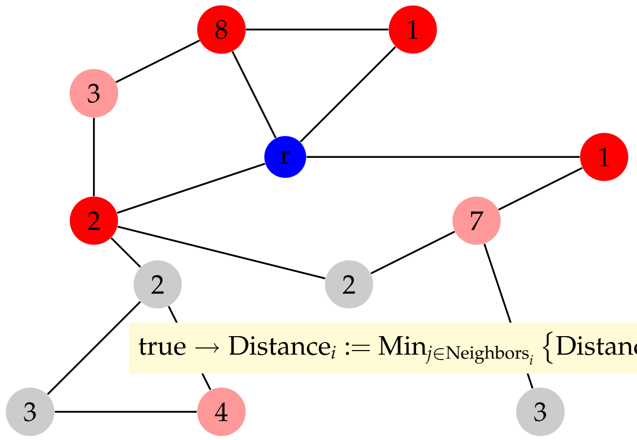

$$\text{true} \rightarrow \text{Distance}_i := \text{Min}_{j\in\text{Neighbors}_i}\{\text{Distance}_j + 1\}$$

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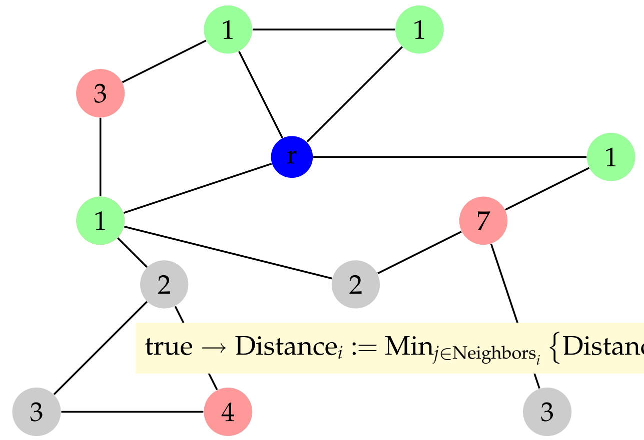

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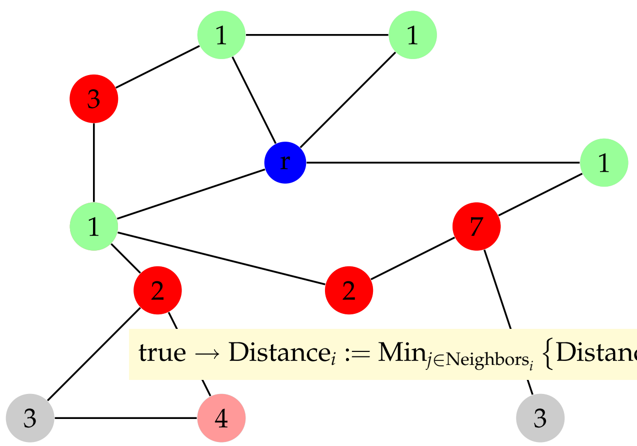

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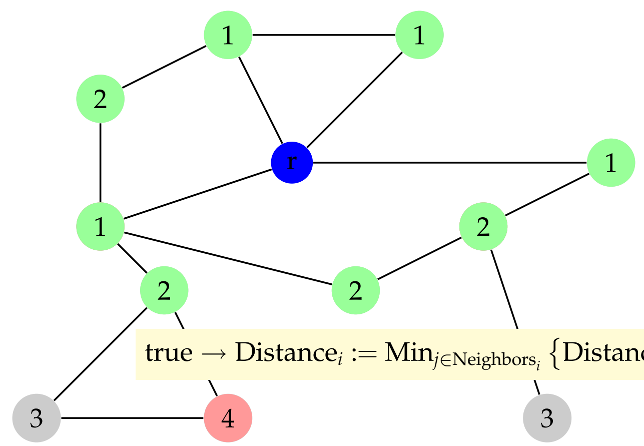

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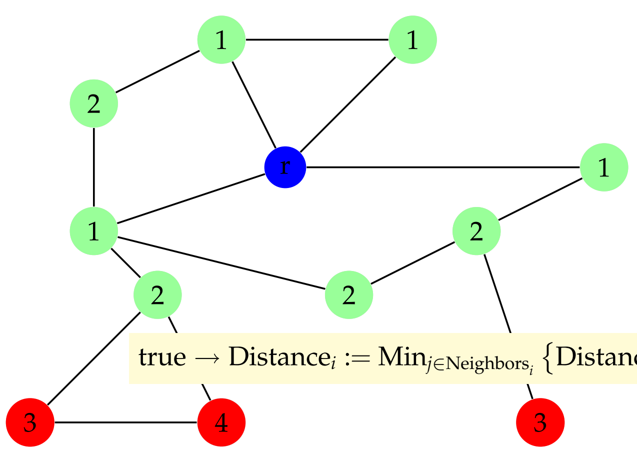

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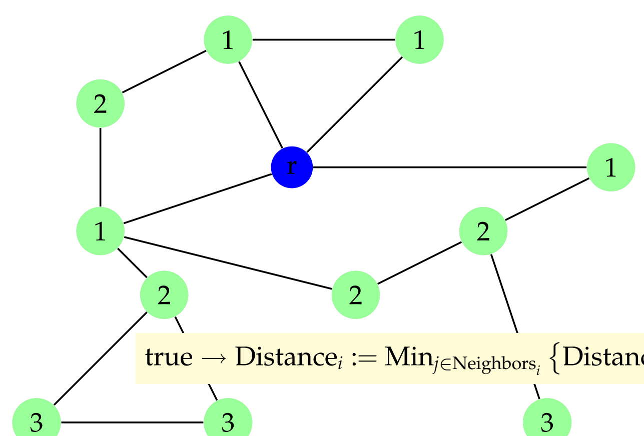

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Scheduling

Definition (Scheduler *a.k.a.* Daemon)

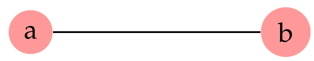
The daemon chooses among activatable processors those that will execute their actions.

- The daemon can be seen as an adversary whose role is to prevent stabilization

Spatial Scheduling

$$\text{true} \rightarrow \text{color}_i := \text{Min}\left\{\Delta \setminus \{\text{color}_j | j \in \text{Neighbors}_i\}\right\}$$

$\Delta = \{0, 1\}$



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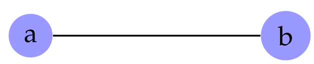

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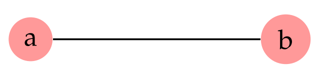

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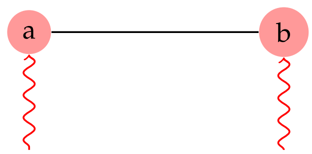

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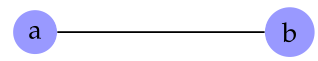

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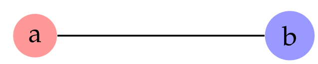

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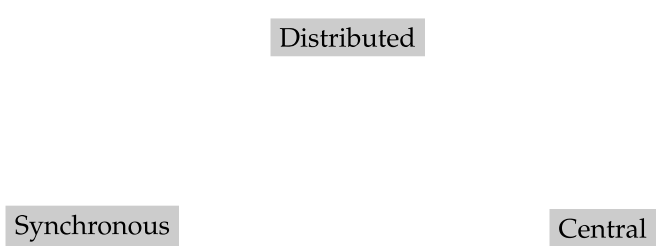

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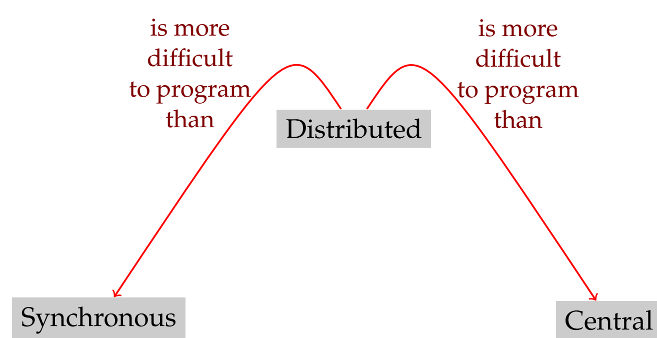

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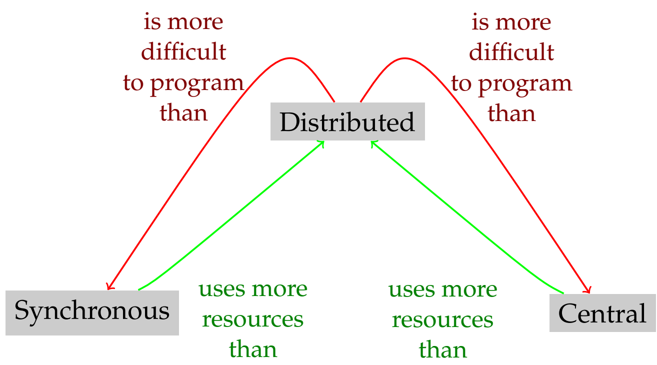
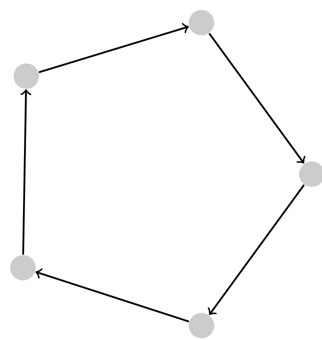


Temporal Scheduling

token → pass token to left neighbor with probability $\frac{1}{2}$



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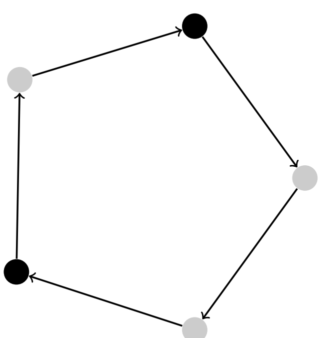

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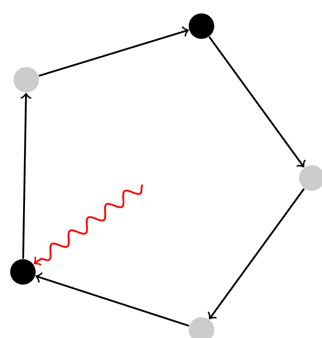

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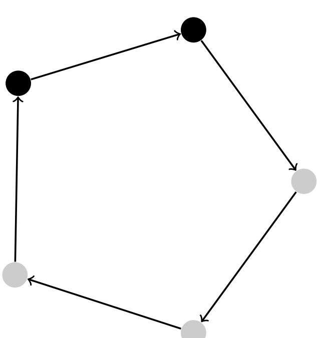

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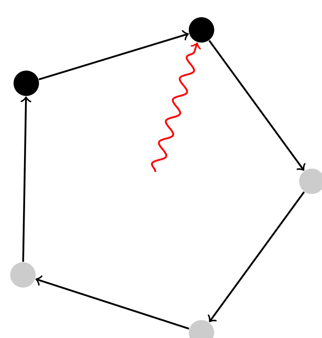

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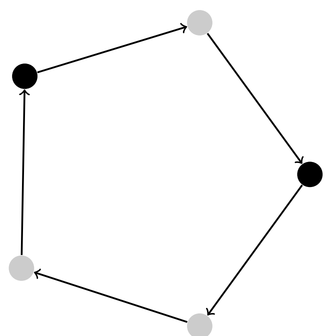

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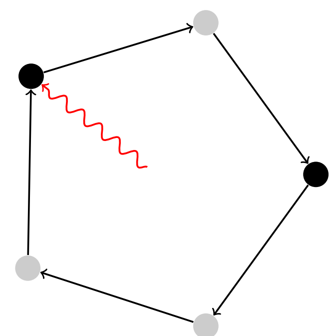

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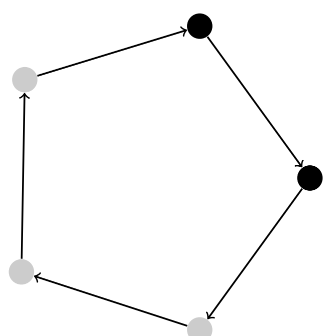

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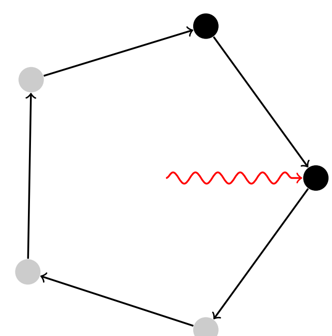

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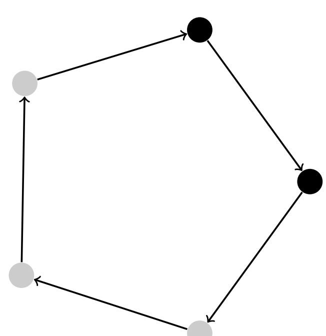

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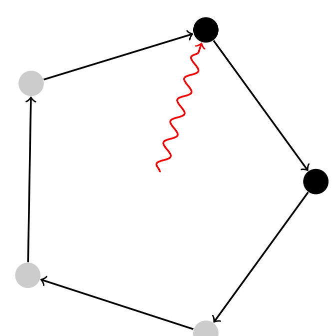

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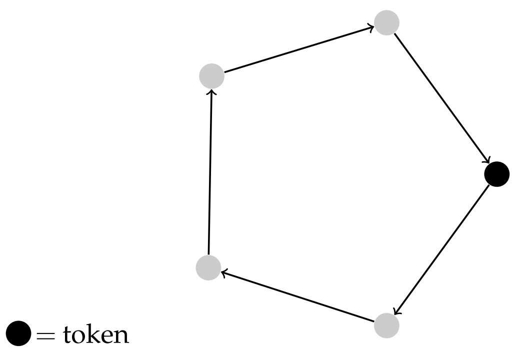

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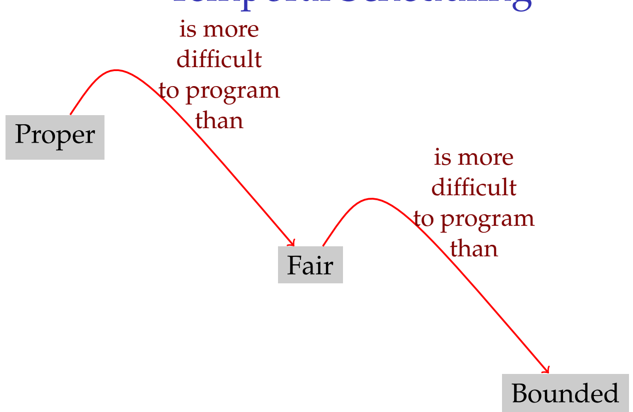

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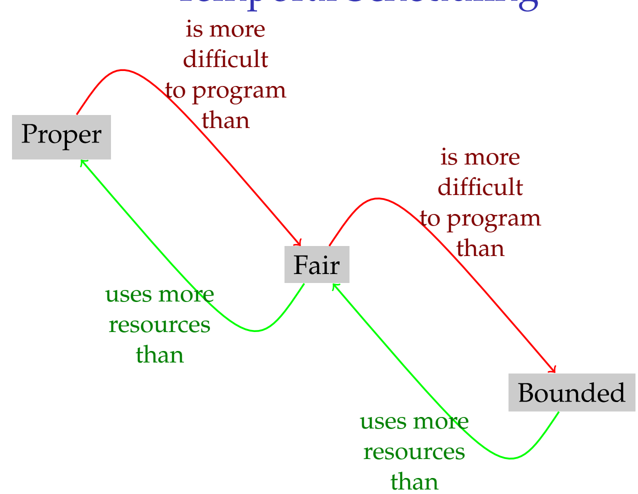

Fair Composition

Basic idea

- Compose several self-stabilizing algorithms $Al_1, Al_2, \ldots Al_k$ such that the results of algorithms $Al_1, Al_2, \ldots Al_i$ can be reused by Al_{i+1}
- Al_{i+1} can not detect whether algorithms $Al_1, Al_2, \ldots Al_i$ have stabilized, but behaves as if

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Example with $k = 2$

- Two simple algorithms server and client are combined to obtain a more complex algorithm
- The server algorithm ensures that some properties (used by the client) will be eventually verified

Example

- Assume the server algorithm Al_1 solves a task defined by a set of legal executions T_1, and the client algorithm Al_2 solves T_2

A_i

- Let A_i be the set of states of process P_i for Al_1, and let $S_i = A_i \times B_i$ be the set of states of process P_i for Al_2, where anytime P_i executes Al_2, it modifies the B_i part of $A_i \times B_i$

A_i — B_i

Fair Composition

Definition (A-projection)

For a configuration c of $S_1 \times S_2 \times \cdots \times S_n$, the A-projection of c is $(a_1, \dots, a_n)$ of $A_1 \times \cdots \times A_n$

Definition (Conditional Stabilization)

Al_2 is self-stabilizing for task T_2 given task T_1 if any fair computation of Al_2 that has an A-projection in T_1 has a suffix in T_2

Fair Composition

Definition (Fair composition)

Al is a fair composition of Al_1 and Al_2 if, in Al, every process alternatively executes actions of Al_1 and Al_2

Theorem

If Al_2 is self-stabilizing for T_2 given T_1, and if Al_1 is self-stabilizing for T_1, then the fair composition of Al_1 and Al_2 is self-stabilizing for T_2

$A_i \rightarrow B_i$

Al_1 Al_2

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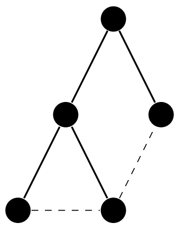
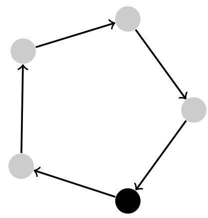
Al_1 Al_2

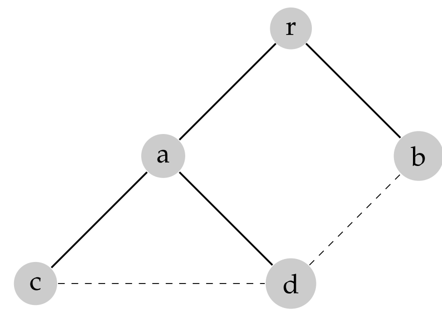
Example

- We are given two self-stabilizing algorithms, one for constructing a tree, one for mutual exclusion on a ring
- We wish to construct a self-stabilizing mutual exclusion algorithm on general graphs

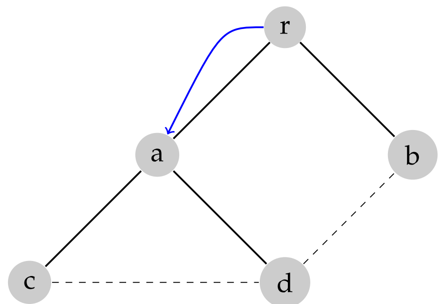




Example



Example



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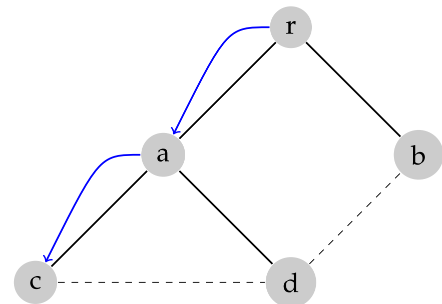

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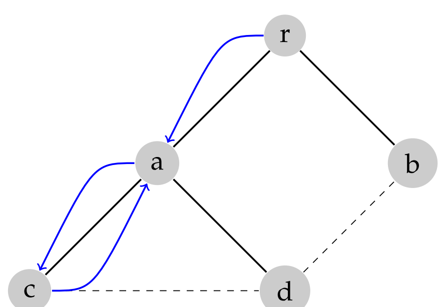

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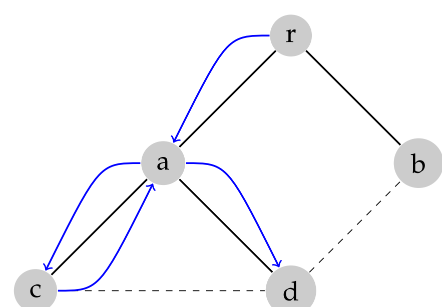
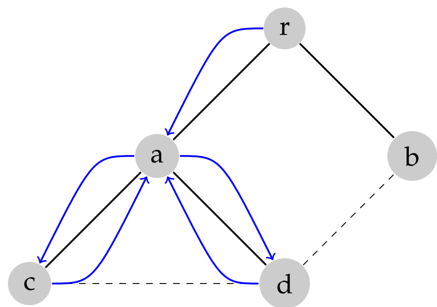


Example



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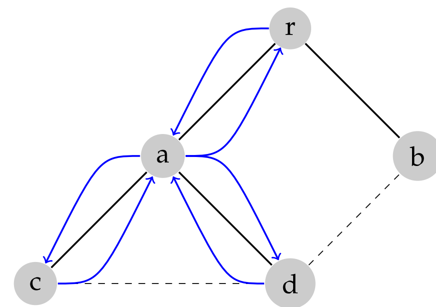

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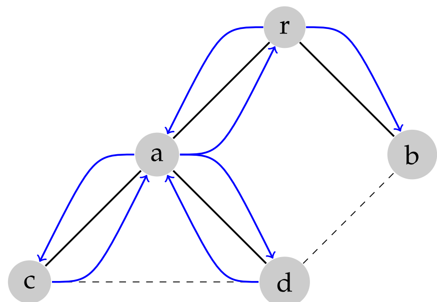
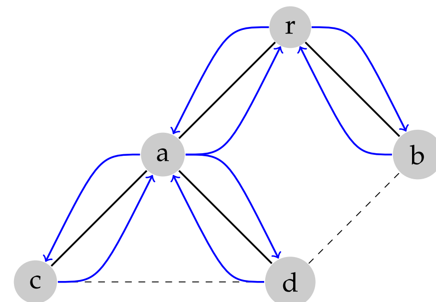
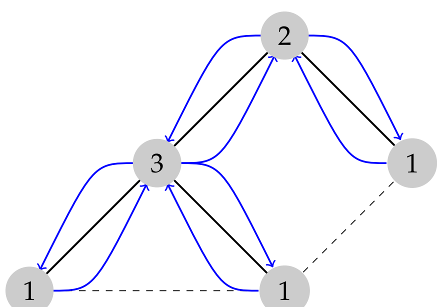


Example



Example



Crosssover Composition

Basic Idea

- We are given two algorithms Al_1 and Al_2
- Al_1 is correct with hypothesis H_1 and Al_2 is correct with hypothesis H_2
- H_2 is more restrictive than H_1

Definition

Crossover Composition The crossover composition is such that Al_2 is conditionnaly executed (only when Al_1 is executed) Al_2 is then correct with hypothesis H_1

Example

Uniform Unidirectionnal ring

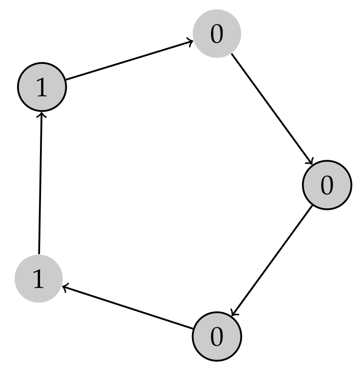
- Each node i has a variable v_i
- Each node i has a token if $v_i \neq v_{i-1} + 1 \mod SND(n)$
- Each node i passes a token by executing $v_i := v_{i-1} + 1 \mod SND(n)$

($SND(n)$: smallest non divisor of n)

Self-stabilization Hypothesis Composition Proof Techniques Conclusion

Example

- $SND(n) = 2$



Self-stabilization Hypothesis Composition Proof Techniques Conclusion

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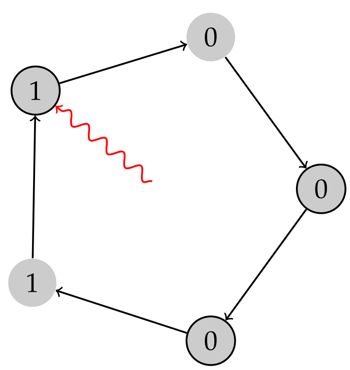

Self-stabilization Hypothesis Composition Proof Techniques Conclusion

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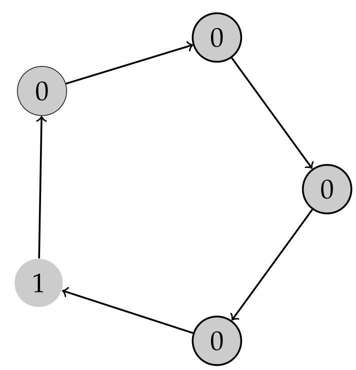

Self-stabilization Hypothesis Composition Proof Techniques Conclusion

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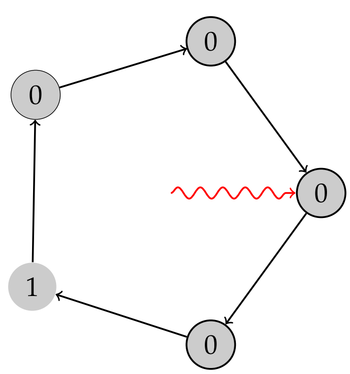

Self-stabilization Hypothesis Composition Proof Techniques Conclusion

Example

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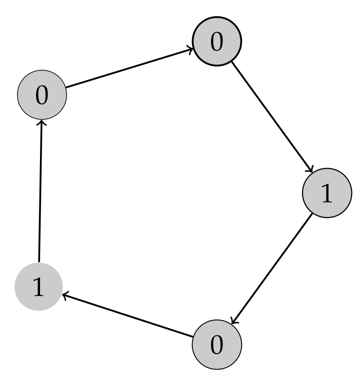


Example

Algorithm Al_1

- A node with the token is activatable
- An activated node always transmit the token
- Al_1 solves the token passing problem with an arbitrary distributed deamon (Hypothesis H_1)

Example

Algorithm Al_2

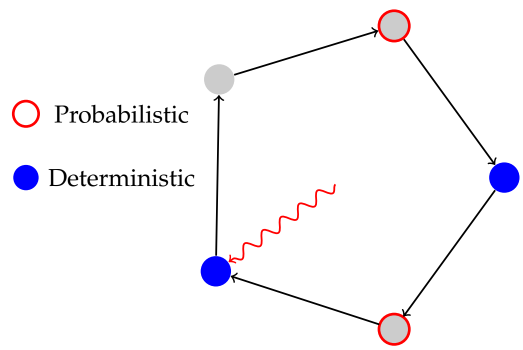
- Each node with a token is activatable
- Each activated node transmists the token with probability $\frac{1}{2}$
- Al_2 solves the self-stabilizing mutual exclusion problem using token passing and a bounded daemon (Hypothesis H_2)

Example

Algorithm Al_2 composed with Al_1

- A node may have two tokens (one deterministic and one probabilistic)
- A node with a deterministic token is activatable
- An activated node passes the deterministic token, and (if it has it) the probabilistic token with probability $\frac{1}{2}$

Example



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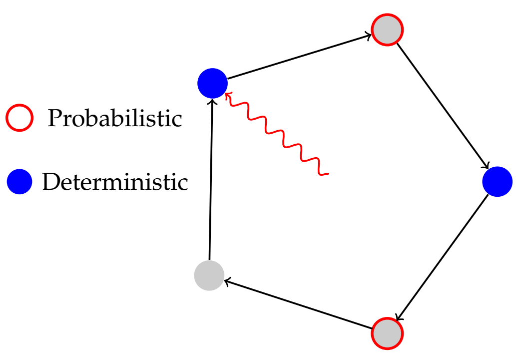

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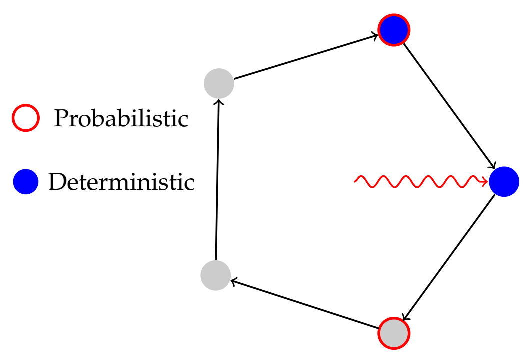

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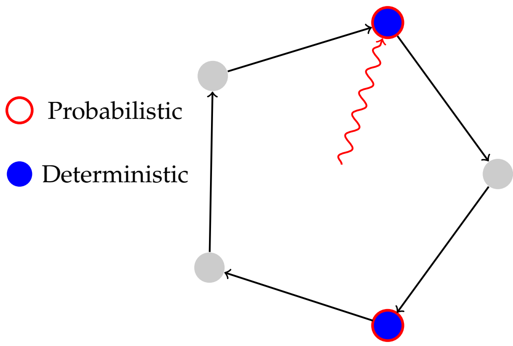

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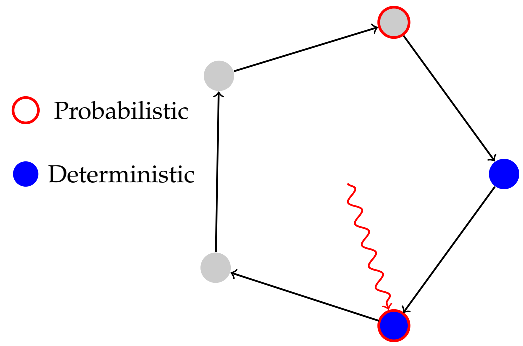

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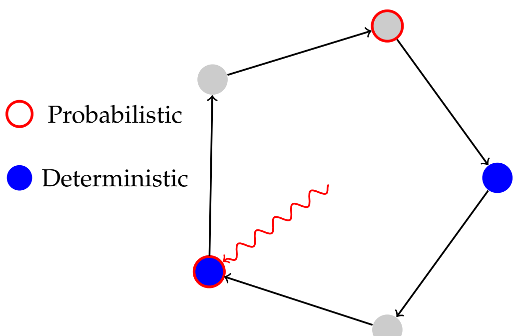

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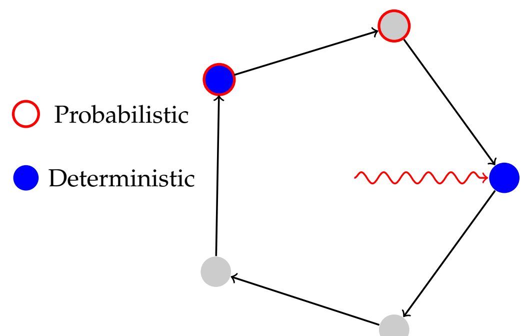

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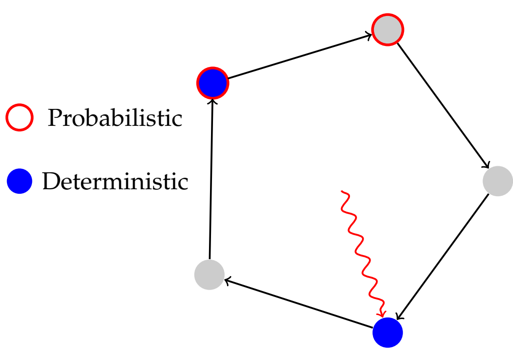

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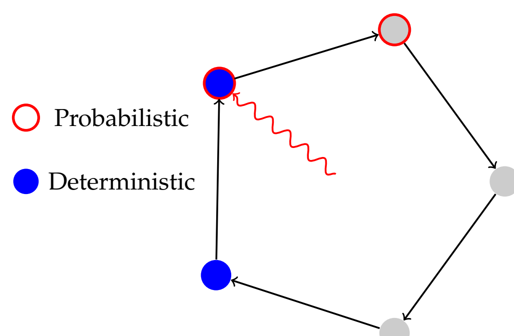

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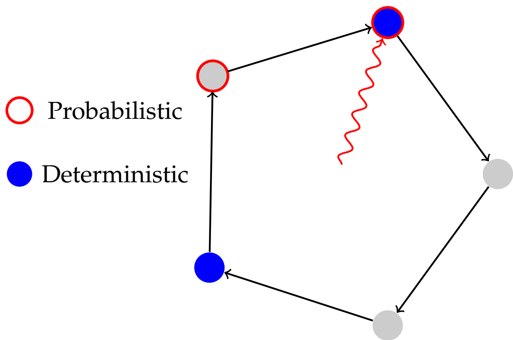

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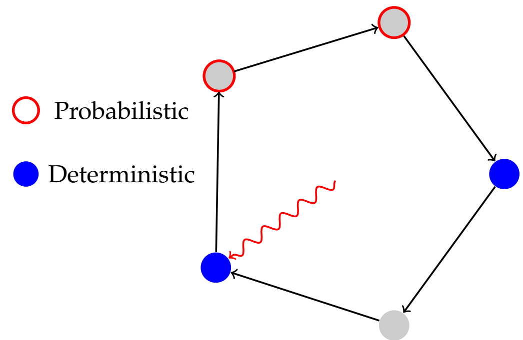

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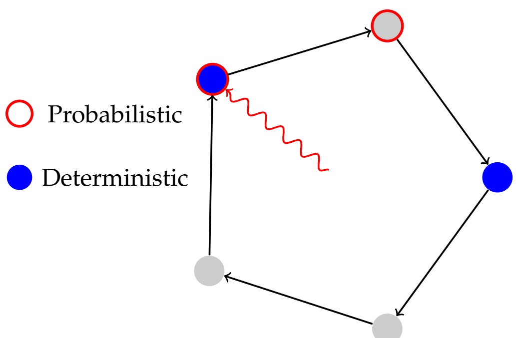

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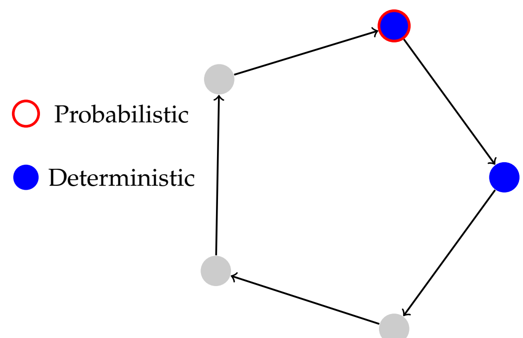


Example

- Al_2 composed with Al_1 solves the self-stabilizing mutual exclusion problem with an arbitrary distributed deamon (H_1)
- Al_2 composed with Al_1 does not solve a more complex problem, but handles less restrictive hypothesis

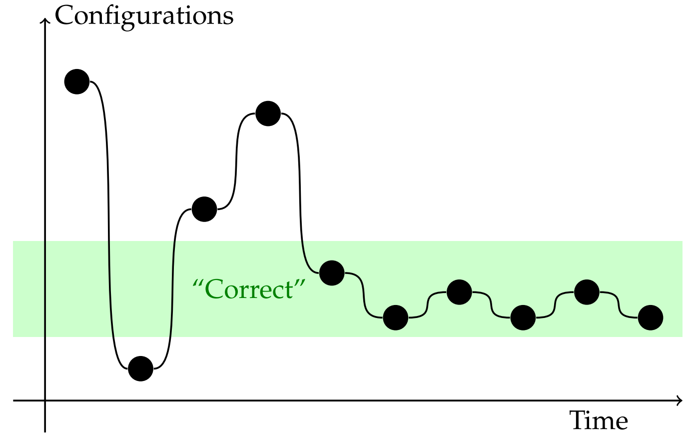
Transfer Function

Basic Idea

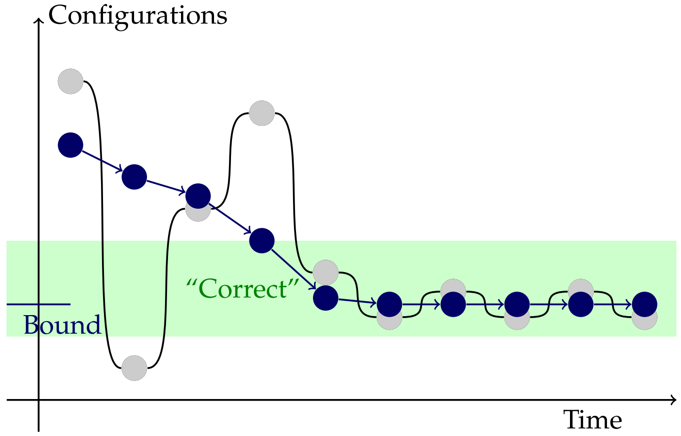
- $c_1 \rightarrow c_2 \rightarrow c_3 \rightarrow c_4 \rightarrow \cdots \rightarrow c_i$
- $FP(c_1) > FP(c_2) > FP(c_3) > \ldots > FP(c_i) = \text{bound}$

- Used to prove convergence
- Can be used to compute the number of steps to reach a legitimate configuration

Transfer Function

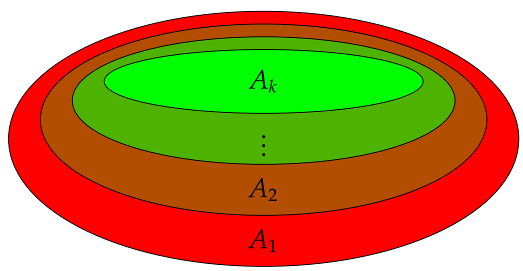


Transfer Function



Convergence stairs

- A_i is a predicate
- A_k is legitimate
- For any i between 1 and k, A_{i+1} is a refinement of A_i



Self-stabilization

Pros

- The network need not be initialized
- When a fault is diagnosed, it is sufficient to identify, then remove or restart the faulty components
- The self-stabilization property does not depend on the nature of the fault
- The self-stabilization property does not depend on the extent of the fault

Self-stabilization

Cons

- "Eventually" does not give any bound on the stabilization time
- A single failure may trigger a correcting action at every node in the network
- Faults must be sufficiently rare that they can be considered are transient
- Nodes never know whether the system is stabilized or not